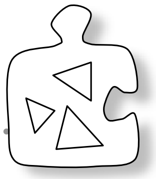
4.3.1 How do shapes change?

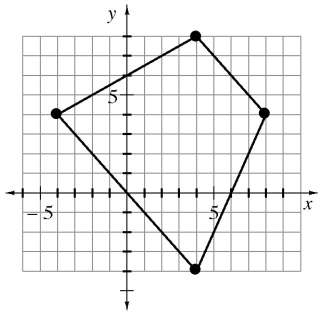


Dilations and Similar Figures

Have you ever wondered how different mirrors work? Most mirrors show you a reflection that looks just like you. But other mirrors, like those commonly found at carnivals and amusement parks, reflect back a face that is stretched or squished. You may look taller, shorter, fatter or skinnier. These effects can be created on the computer if you put a picture into a photo program. If you do not follow the mathematical principles of proportionality when you enlarge or shrink a photo, you may find that the picture is stretched thin or spread out, and not at all like the original. Today you will look at enlarging and reducing shapes using dilations to explore why a shape changes in certain ways.

4-69. UNDOING DILATION

In Chapter 2, you looked at dilations and multiplied each of the coordinates of a shape to change its size. How can you undo dilation?



Charlie multiplied each coordinate of the vertices of a shape by 4 to create the dilated shape at right.

a. If Charlie multiplied to find this shape, what operation would undo his dilation? Why?

b. On a Lesson 4.3.1A Resource Page, undo the dilation on the graph above. Label the vertices of Charlie's original shape. How does the shape compare to the dilated shape?

4-70. Alana was also working with dilations. She wondered, *"What would happen if I multiplied each coordinate of a shape by $\frac{1}{3}$?"* On the Lesson 4.3.1A Resource Page, graph and connect the points below in order to form her dilated shape.

$(-1,-1)$ $(-1,1)$ $(1,2)$ $(2,-1)$

a. Alana graphed this shape by multiplying each of her original coordinates by $\frac{1}{3}$. What do you think Alana's shape looked like before the dilation? Make a prediction.

b. On the same graph, undo the dilation to show Alana's original shape. List the coordinates of the vertices of Alana's original shape.

c. What did you do to each coordinate to undo the dilation? How did the shape change?

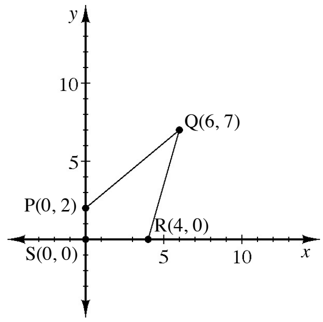
d. Why do you think the shape changed in this way?

4-71. With your team, look carefully at Alana's dilated and original shapes and describe how the two shapes are related. Use the questions below to help you.

- How are the sides of the small and large shape related?
- How many of the small sides does it take to measure the **corresponding** (matching) side of the large shape? Is this true for all of the sides?
- Compare the four angles of the smaller shape to those of the larger shape. What can you say for sure about one matching pair of these angles? What appears to be true about the other three pairs?

4-72. CHANGING SHAPE

When you multiplied each coordinate of a shape by the same constant, you saw that sometimes the shape became smaller and sometimes it became larger. In Chapter 2, you moved shapes on a graph without changing their size or shape by rotating, reflecting and translating them. What other ways can you change a shape?



Your task: Work with your team to predict what you could do to the coordinates of the shape above to make it look stretched or squished, and what actions will keep the shape the same. Use the questions below to guide your discussion.

Discussion Points

What do you think will change if both the x- and y-coordinates of the points P, Q, R, and S are multiplied by the same number, such as 4?

What do you think will happen if only the x-coordinates are multiplied by 3?

What do you think will happen if just the y-coordinates are multiplied by 2?

What do you think will happen if the x- and y-coordinates are multiplied by different numbers, like 2 for x and 3 for y?

4-73. Test the predictions your team made in problem 4-72. On the Lesson 4.3.1B Resource Page, graph each of the shapes described below.

a. Dilate each coordinate of shape $PQRS$ by multiplying each x-coordinate and each y-coordinate by 4. Graph the dilated shape on the same graph using a color other than black.

b. Go back to the original shape and this time multiply only the x-coordinates by 3. Leave the y-coordinates the same. Find, graph, and connect the new coordinates.

c. What happened to the shape? Why did this happen?

d. Look at the predictions your team made in problem 4-72. Do you still agree with your predictions? Revise your predictions if necessary, based on your work so far. What do you think will happen if you multiply only the y-coordinates of the vertices by a number? Be ready to explain your **reasoning**.

4-74. **Similar figures** are figures that have the same shape, but not necessarily the same size. One characteristic of similar shapes is that the sides of one shape are each the same number of times bigger than the corresponding sides of the smaller shape.

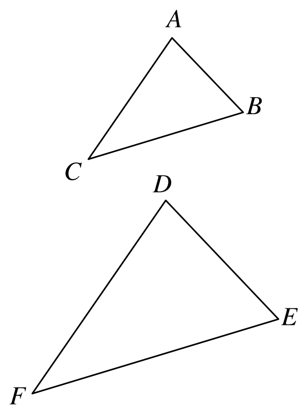
Which pairs of shapes that you have worked with in this lesson are similar and which are not? **Justify** your answer using specific examples.

METHODS AND MEANINGS

MATH NOTES

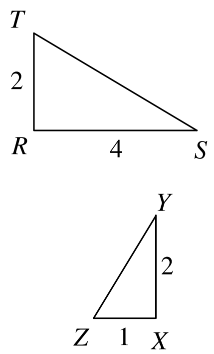
Corresponding Parts of Similar Shapes

Two figures are **similar** if they have the same shape but not necessarily the same size. For example, all semi-circles are similar as are all squares, no matter how they are oriented.



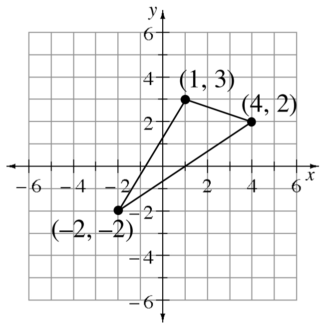
In order to check whether figures are similar, you need to decide which parts of one figure **correspond** (match up) to which parts of the other. For example, in the triangles at right, triangle *DEF* is a dilation of triangle *ABC* where side *AB* is dilated to get side *DE*, side *AC* is dilated to get side *DF* and side *BC* is dilated to get side *EF*. We say that side *AB* **corresponds** to side *DE* or that they are **corresponding sides**. Notice that vertex *A* corresponds to vertex *D*, *C* to *F*, and *B* to *E*.

Not all correspondences are so easily seen. Sometimes you have to rotate or reflect the shapes in your mind so that you can tell which parts are the corresponding sides, angles, or vertices. For example, the two triangles at right are similar with *R* corresponding to *X*, *S* to *Y*, and *T* to *Z*. We can get triangle *XYZ* from triangle *RST* by a dilation of $\frac{1}{2}$ followed by a 90° counter-clockwise (↺) turn.



Shapes that are similar and have the same size are called **congruent**. Congruent shapes have corresponding sides of equal length and corresponding angles that have equal measure.

4-75. Create a large coordinate grid on graph paper and graph the triangle at right. Multiply the y-coordinate of each point by 4. Then graph the new shape. Make sure you connect your points. List the points for the new shape. Are the two figures similar? Why or why not?

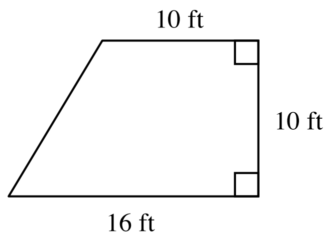


4-76. Kris said, "The Rawlings Rockets basketball team does not have any really tall players." These are the player's heights in inches: 70, 77, 75, 68, 88, 70, and 72.

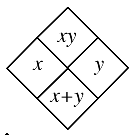
a. Which number does not seem to fit this set of data?

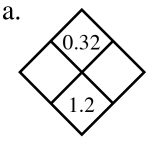
b. Do you agree or disagree with Kris? Explain.

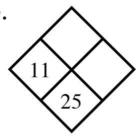
4-77. Dante is ordering wood flooring for his bedroom, which is shaped like a trapezoid (shown at right). If the flooring materials will cost $5 per square foot, how much should he expect to pay for the materials?

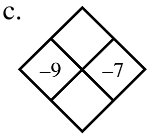


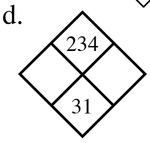
4-78. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a. 

b. 

c. 

d. 

4-79. Darnell is designing a new game. He will have 110 different colored blocks in a bag. While a person is blindfolded, they will reach in and pull out a block. The color of the block determines the prize according to Darnell's sign at right.

blue → small toy
purple → hat
green → large stuffed animal

a. If he wants players to have a 60% probability of winning a small toy, how many blue blocks should he have?

b. If he wants players to have a 10% probability of winning a large stuffed animal, how many green blocks should he have?

4.3.2 Are they similar?

Identifying Similar Shapes

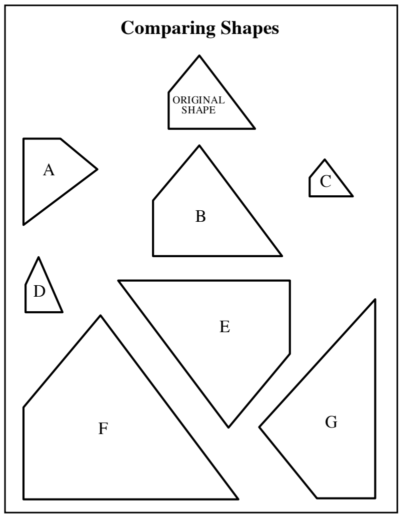
Have you ever noticed how many different kinds of cell phones there are? Sometimes you might have a cell phone that is similar to one of your friends' cell phones because it is the same brand, but it might be a different model or color. Occasionally, two people will have the exact same cell phone, including brand, model and color. Sorting objects into groups based on their sameness is also done in math. As you work with your team to sort shapes, ask the following questions:

How do the shapes grow or shrink?

What parts can we compare?

How can we write the comparison?

4-80. WHICH SHAPES ARE SIMILAR?



If two shapes appear to have the same **general** relationship between sides, how can you decide for sure if those shapes are similar? Work with your team to:

- Carefully cut out the original shape and shapes A-G from one Lesson 4.3.2 Resource Page.
- Decide how each shape is related to the original shape. (Each person should use an uncut copy of the resource page and the team's cut out shapes to help.)
- Compare the angles and the sides of shapes A-G to the original shape.

a. Which shapes are similar to the original shape? Give specific reasons to **justify** your conclusions.

b. Look only at the shapes that are similar to the original shape. What do these shapes have in common? What is different about them? Be specific.

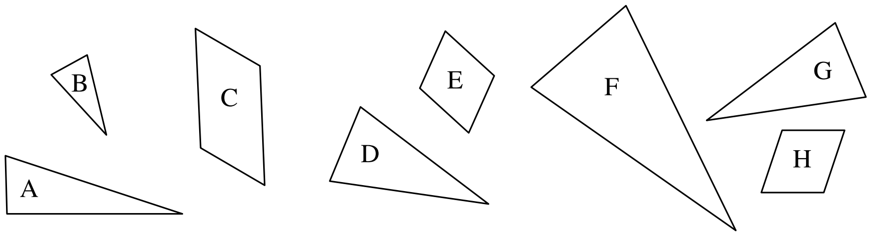
c. When two shapes are similar, the **scale factor** is the number you multiply the length of the side of one shape by to get the length of the corresponding side of the new shape. What is the scale factor between the original shape and shape E? Is each side of the shape enlarged the same number of times? Use a ruler to help you decide.

d. What is the scale factor between the original shape and shape C? Why is it less than 1?

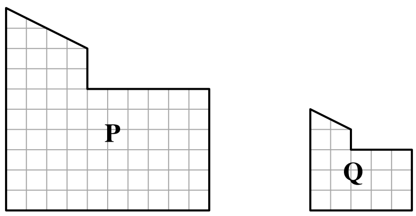
4-81. Which shape from problem 4-80 is exactly equal to the original shape in every way? Shapes like this that are similar but do not grow or shrink are called **congruent** shapes.

a. Record the pairs of shapes below that appear to be congruent to each other.

b. Get a piece of tracing paper from your teacher and use it to check that the shapes you identified as congruent have exactly the same size and shape. Were you correct? If not, why not? Write your answers in complete sentences.



4-82. Quan enlarged shape Q to make shape P, below. Are his shapes similar? If they are similar, identify the scale factor (multiplier). If they are not, demonstrate that at least one pair of sides does not share the scale factor.



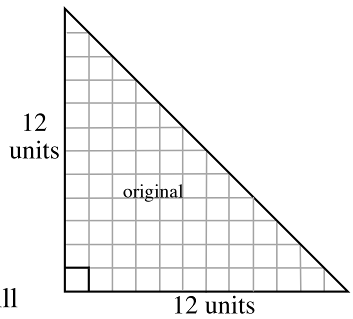
4-83. Draw each of the shapes in problem 4-82 on graph paper. Color code the corresponding sides on each shape using the colors suggested by your teacher.

a. Compare the green sides of each shape. What do you notice about those sides?

b. Compare each of the other five sides of shape P with their corresponding sides on shape Q. What do you notice about those pairs of sides?

c. Imagine enlarging shape P to make a new shape R that has a base that is 25 units long. If shape R is similar to P, predict the length of the blue side of shape R *without drawing the shape*.

4-84. Using the triangle shown at right as the original figure, *predict* which of the scale factors below would enlarge (make bigger) or reduce (make smaller) the shape. (Do not actually make a new shape.)



$\frac{5}{3}$ $\quad\frac{3}{4}$ $\quad\frac{7}{6}$ $\quad\frac{2}{3}$

After you write down your prediction, decide which scale factor each member of your team will use. Then copy the original figure on graph paper and draw a similar triangle using your scale factor.

a. Show your new triangle to your teammates and check your predictions. Which scale factors made the triangle larger? Which made the triangle smaller? Is there a pattern?

b. Which parts of the new triangles remained the same as the original triangle? Which parts changed? How do you know?

c. Each of the new triangles is similar to the original triangle used to create it. Compare the corresponding (matching) sides and angles to each other. Describe the relationship or explain why you think there is no relationship.

d. What scale factor could you use to create a triangle that is congruent (identical) to the original? Explain.

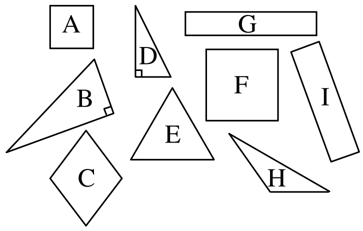
e. **Additional Challenge:** Find a scale factor that is less than 2 that will make a similar shape that is larger than the original.

4-85. LEARNING LOG

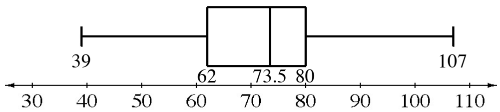
In your Learning Log, explain how to determine when shapes are similar. In order to decide if two shapes are similar, what do you need to know about the side length? The angles? Title your entry "Finding Similar Shapes" and include today's date.

Review & Preview

4-86. Which of the shapes at right appear to be similar? Explain how you know.



4-87. Mt. Rose Middle School collected canned food to donate to a local charity. Each classroom kept track of how many cans it collected. The number of cans in each room were: 107, 55, 39, 79, 86, 62, 65, 70, 80, and 77. The principal displayed the data in the box-and-whisker plot below.

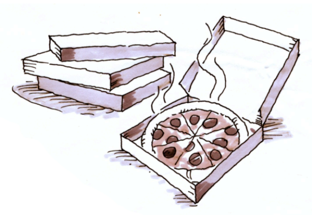


a. What is the range of the data? Are there any outliers?

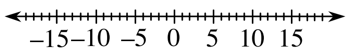
b. The main office staff collected 55 cans, the counseling staff collected 74 cans, and the custodial staff collected 67 cans.

On grid paper, make a new box-and-whisker plot that includes this data. Clearly label the median and the upper and lower quartiles.

4-88. For the end of year party, Mt. Rose Middle School ordered 134 pizzas. There were eight fewer combo pizzas than there were pepperoni pizzas. There were three times as many combo pizzas as cheese pizzas. Use the 5-D Process to determine how many of each kind of pizza were ordered.



4-89. Copy each part below on your paper, then use the number line to help you fill in < (less than) or > (greater than) on the blank line.



a. -5 __ -2 b. 8 __ -1 c. -5 __ 0 d. -15 __ -14

4-90. Simplify the following expressions using the order of operations.

a. $7 \cdot 8 - 4(6-2) + 18$

b. $6^2 - (8 \cdot 3) + 2^2(7 \cdot 3)$

c. $\frac{14}{2} - 3(8-6) + 7^2$

d. $-9 - 3(7-2) + \frac{24}{3}$

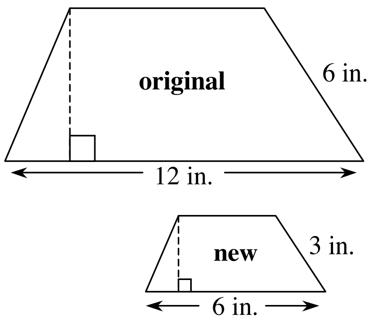
4.3.3 What do similar shapes tell us?

Working With Corresponding Sides

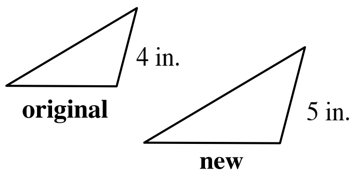
Sometimes graphic artists have a shape that they need to make larger to use for a sign or make smaller to use for a bumper sticker. They have to be sure that the shapes look the same no matter what size they are. How do artists know what the side length of a similar shape should be, if it needs to be larger or smaller than the original?

4-91. With your team, find the scale factor between each pair of similar shapes. That is, what are the sides of each original shape multiplied by to get the new shape? Assume shapes are drawn to scale.

a.



b.



4-92. It may have been easier to recognize the scale factor between the two shapes in part (a) of problem 4-91 than it was to determine the scale factor between the two shapes in part (b). When sides are not even multiples of each other (like the sides labeled 4 in. and 5 in. in part (b)), it is useful to have another **strategy** for finding the scale factor.

Your task: Work with your team to describe a **strategy** for finding the scale factor between any two shapes. Refer to the questions below to begin your discussion.

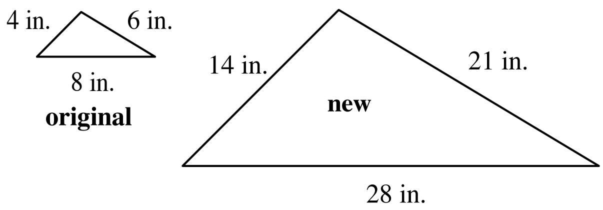
Discussion Points

How can we use pairs of corresponding sides to write the scale factor?

Will the scale factor between the shapes be more or less than one?

Does it matter which pair of corresponding sides we use?

4-93. A study team was working together to find the scale factor for the two similar triangles at right.



- Claudia set up the ratio $\frac{14}{4}$ to find the scale factor.
- Issac set up the ratio $\frac{28}{8}$ to find the scale factor.
- Paula set up the ratio $\frac{21}{6}$ to find the scale factor.

a. What did the students do differently when they found their scale factors?

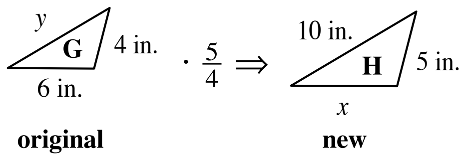
b. Do the triangles have more than one scale factor? If not, show how they are the same.

c. Why does it make sense that the ratios are equal?

d. The different scale factors for this triangle form a **family of fractions**. In a fraction family, each of the fractions is equivalent to the others and can be related to the others using a Giant One. The fraction with the smallest numerator and denominator (that are still integers) in this family is the **root fraction**. For example, in the fraction family that includes $\frac{3}{6}$ and $\frac{5}{10}$, the root fraction is $\frac{1}{2}$. You have probably worked with root fractions before when you were reducing fractions to their lowest terms.

Are any of the scale factors the team wrote a root fraction for this family? If so, give **reasons** to explain how you know. If not, find the root fraction.

4-94. Alex was working with the two triangles from problem 4-91, but he now has a few more pieces of information about the sides. He has represented the new information and his scale factor in the diagram reprinted at right. Sketch his diagram on your own paper.



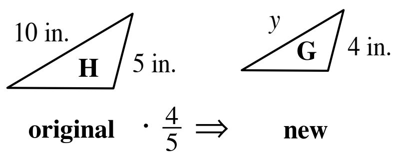
a. Use the scale factor to find the length of the side labeled x. Show your work.

b. Since Alex multiplied the side lengths of triangle G to get triangle H, he needs to undo the enlargement to find the side labeled y. What math operation would he use to undo the enlargement? Write an expression and be prepared to explain your **reasoning**. If you are able, simplify the expression to find y.

c. If triangle H had been the original triangle and triangle G had been the new triangle, how would the scale factor change? What would the new scale factor be? Explain.

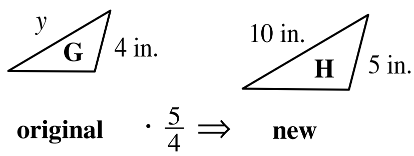
4-95. Alex and Maria were trying to find the side labeled y in problem 4-94. Their work is below.

Maria: *"I made H the original triangle, then multiplied H by $\frac{4}{5}$. The side marked 10 inches became only 8 inches."*



$10 \text{ in.} \cdot \frac{4}{5} = 8 \text{ in.}$

Alex: *"To undo the change, I divided 10 inches by the scale factor ($\frac{5}{4}$), and that side of the triangle got shorter."*



$10 \text{ in.} \div \frac{5}{4} = 8 \text{ in.}$

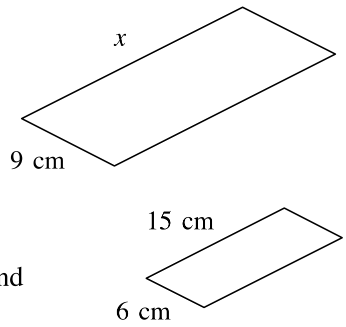
a. Compare the way Alex and Maria solved for y. Why did Alex divide and Maria multiply?

b. Compare their scale factors. Why did Maria multiply by $\frac{4}{5}$?

Problem continues on next page. →

4-95. *Problem continued from previous page.*

c. Consider the pair of similar parallelograms below. For these shapes, Alex found the scale factor $\frac{\text{new}}{\text{original}} = \frac{6}{9}$ and used it to write the expression $15 \div \frac{6}{9}$ to find x.

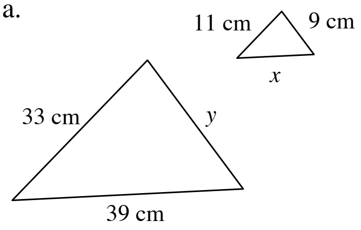


Use Maria's strategy to rewrite the problem to find x.

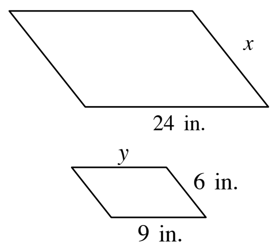
- Which shape would she label "new" and which would she label "original"?
- What scale factor would she use?
- What does x equal?

4-96. For the pairs of similar shapes below, find the lengths of the missing sides. Be sure to show your calculation. You can choose which shape is "new" and which is "original" in each pair. Assume the shapes are all drawn to scale. The shapes in part (b) are parallelograms.

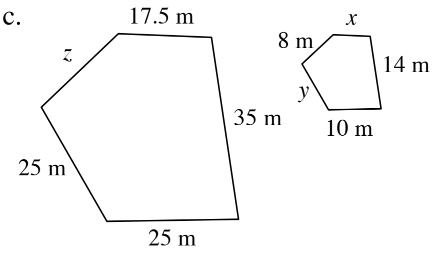
a.



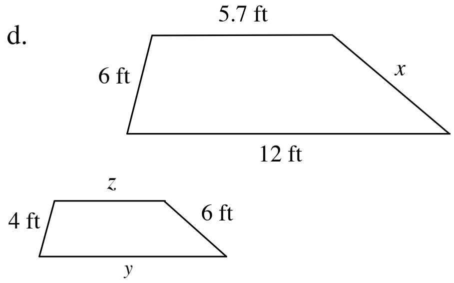
b.



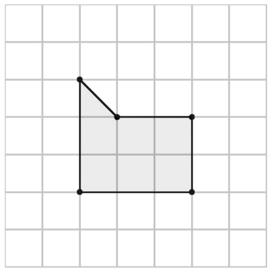
c.



d.



4-97. **Additional Challenge:** Copy the figure shown at right on graph paper.

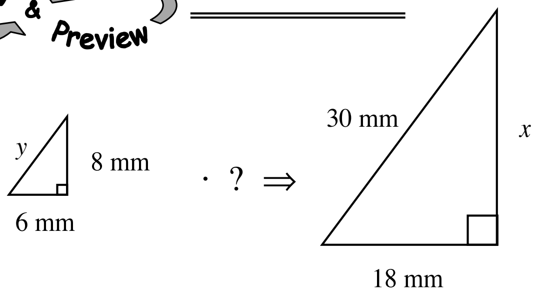


a. Find the area of the shape.

b. Enlarge the shape by a scale factor of 2, and draw the new shape. Find the area.

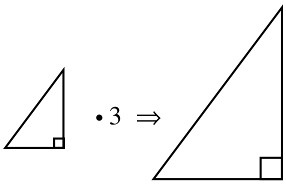
Review & Preview

4-98. Sketch the two similar triangles at right on your own paper. Find the scale factor and the missing side lengths.



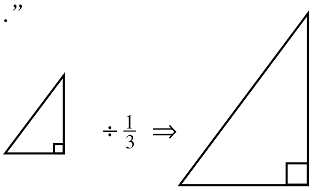
4-99. Alex and Maria were trying to find the side labeled x in problem 4-98. Their work is shown below.

Alex: *"I noticed that when I multiplied by 3, the sides of the triangle got longer."*



$8 \cdot 3 = 24$

Maria: *"I remember that when we were dilating shapes in Lesson 4.3.1, my shape got bigger when I divided by* $\frac{1}{3}$*."*

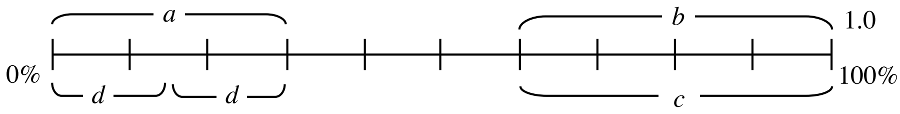


$8 \div \frac{1}{3} = 24$

a. Look at each student's work. Why do both multiplying by 3 and dividing by $\frac{1}{3}$ make the triangles larger?

b. Use Alex and Maria's **strategy** to write two expressions to find the value of y in problem 4-98.

4-100. Find the missing lengths or values on the diagram below. Assume that the line is evenly divided.



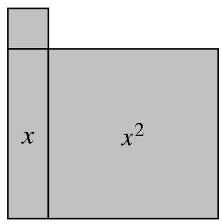
4-101. Use graph paper to solve the following problem.

a. Draw an xy-coordinate graph and label each axis. Plot the following ordered pairs: $(2, 3), (-2, 3), (-2, -3), (2, -3)$. Connect the points in the order given as you plot them, then connect the fourth point to the first one.

b. Describe the shape on your graph. What is its area? What is its perimeter?

c. Change only two points so that the shape has an area of 32 square units. List your points. Is there more than one answer?

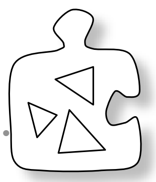
4-102. Sketch the algebra tile shape at right on your paper. Write an expression for the perimeter, then find the perimeter for each of the given values of x.



a. $x = 7$ cm

b. $x = 5.5$ cm

4.3.4 How do I find a missing side?



Solving Problems Involving Similar Shapes

Architects create scaled plans for building houses, artists use sketches to plan murals for the sides of buildings, and companies create smaller sizes of their products for display in stores. Each of these models is created to show all of the information about the “real” object, without being the actual size of the object. Today you will work with your team to find **strategies** that you can use when you are missing some of the information about a set of similar shapes. As you work, look for more than one way to solve the problem.

4-103. MODEL TRAINS

Kenen loves trains, especially those that run on narrow gauge tracks (the gauge of a track measures how far apart the rails are). He has decided to build a model train of the Rio Grande, a popular narrow gauge train.

Use the following information to help him know how big his model should be:

- The real track has a gauge of 3 feet (36 inches).
- His model railroad track has a gauge of $\frac{3}{4}$ inches.
- The Rio Grande train he wants to model has driving wheels that measure 44 inches high.

Your task: With your team, discuss what you know about the model train Kenen will build. What scale factor should he use? What will be the height of the driving wheels of his model? Be prepared to share your **strategies** with the class.

4-104. Heather and Cindy are playing "Guess My Shape." Heather has to describe a shape to Cindy accurately enough so that Cindy can draw it without ever seeing the shape. Heather gives Cindy these clues:

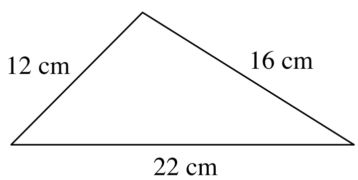
Clue #1: The shape is similar to a rectangle with a base of 7 cm and a height of 4 cm.

Clue #2: The shape is five times larger than the shape it is similar to.

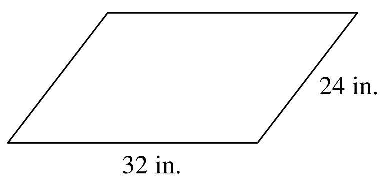
a. Has Heather given Cindy enough information to draw the shape? If so, sketch the shape on your paper. If not, write a question to ask Heather to get the additional information you need.

b. Use what you know about similar shapes to write a set of "Guess My Shape" clues to describe each of the mystery shapes below. Your clues should be complete enough that someone in another class could read them and draw the shape. Be sure to include at least one clue about the relationship between the mystery shape and a similar shape.

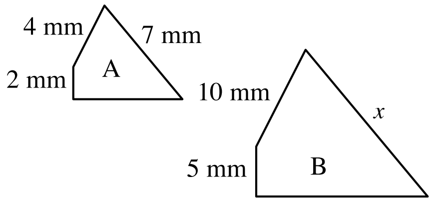
i. SHAPE A: Triangle



ii. SHAPE B: Parallelogram



4-105. Nick enlarged figure A at right so that it became the similar figure B. His diagrams are shown at right.

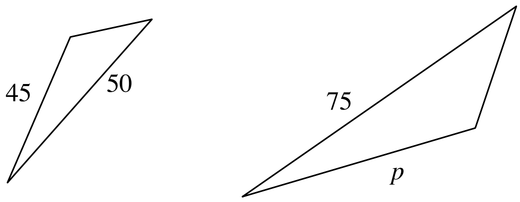


a. Write all the ratios that compare the corresponding sides of figure B to figure A.

b. What is the relationship between these ratios? How do you know?

c. Find two different ways to find the value of x in this quadrilateral. Does your solution seem reasonable? Be ready to share your **strategies** with the class.

4-106. Fatima solved for p in the diagram of similar triangles below and got $p = 30$. Looking at her answer, she knows she made a mistake. What would make Fatima think that her answer is wrong?



4-107. LEARNING LOG

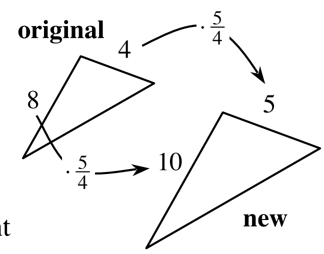
In your Learning Log, write a description about how to find the missing side of a similar shape. Be specific about your **strategy** and include a picture with labels. Put today's date on your entry and title it "Finding Missing Sides of Similar Shapes."

MATH NOTES

METHODS AND MEANINGS

Scale Factor

A **scale factor** is a ratio that describes how two quantities or lengths are related. A scale factor that describes how two similar shapes are related can be found by writing a ratio between any pair of corresponding sides as $\frac{\text{new}}{\text{original}}$.



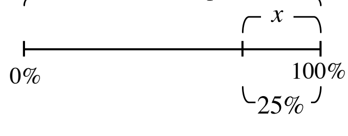
For example, the two similar triangles at right are related by a scale factor of $\frac{5}{4}$ because the side lengths of the new triangle can be found by multiplying the corresponding side lengths of the original triangle by $\frac{5}{4}$.

A scale factor greater than one **enlarges** a shape (makes it larger). A scale factor between zero and one **reduces** a shape (makes it smaller). If a scale factor is equal to one, the two similar shapes are identical and are called **congruent**.

Review & Preview

4-108. The electronics department is having a No Sales Tax Sale! In addition, all of the items in the department are on sale for 25% off. Wyatt is looking at a music player that normally costs $120. He has $95 to spend, and he is wondering if he has enough money to buy it.

a. Wyatt sketched the diagram at right. Use the work he started to find 25% of $120. Is this the price he will pay?

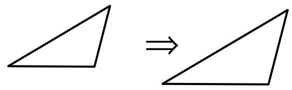


b. Does Wyatt have enough money?

c. Would he have enough money if he had to pay the 5.5% sales tax on the sale price?

4-109. For each expression below:

- Sketch and label a pair of similar shapes (like those at right, or in problem 4-94) that would result in this calculation.



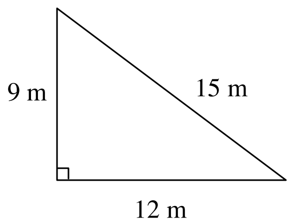
- Rewrite the expression so that the operation is multiplication.
- Calculate the value of the expression.

a. $6 \div \frac{1}{2}$ b. $4 \div \frac{2}{3}$

4-110. A biologist was sitting near a pond and noticed a large number of dragonflies. He also saw both frogs and fish trying to eat the dragonflies. He counted a total of 89 fish, frogs, and dragonflies. He noticed that there were four times as many dragonflies as fish and that the frogs were five more than twice the number of fish.

Use the 5-D Process to determine how many fish, frogs, and dragonflies the biologist counted.

4-111. Sketch the triangle, then redraw it with sides that are $\frac{1}{3}$ as long as the original.

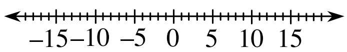


a. Calculate the perimeters of both triangles.

b. Calculate the areas of both triangles.

c. What is the relationship between the perimeters of the triangles?

4-112. Copy the following problems, then use the number line to help you fill in < (less than) or > (greater than) on the blank line.



a. –4 __ –8 b. 7 __ –7 c. –6 __ –5 d. –1 __ 0

Chapter 4 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① SUMMARIZING MY UNDERSTANDING

Data is used every day in the world around us to help people make decisions. Oftentimes when people are trying to make a convincing argument or market a product, graphical displays and measures of center are chosen based on how they can help to make a strong, convincing argument.

Today you will use the skills you learned in this chapter to analyze the talents of two frogs. Look at the data provided about Frog A and Frog B and analyze the information.

Seven best jumps (inches)

Frog A	Frog B
177	177
221	201
224	203
230	230
239	236
240	236
239	257

- Work with your team to display the data in a box-and-whisker plot, a histogram, and a stem-and-leaf plot. Calculate any measures of central tendency that might help you decide which frog is a better jumper.
- After you learn as much as you can about these two frogs, decide which frog you think is the best jumper. Obtain a Summarizing My Understanding Graphic Organizer, choose a data display, and present the statistics that you believe will support your claim.
- Write a convincing argument for why the frog you chose is the best jumper. Make sure to refer to your graphs and measures of central tendency in your argument.

② WHAT HAVE I LEARNED?

Working the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

Solve each problem as completely as you can. The table at the end of this closure section provides answers to these problems. It also tells you where you can find additional help and practice on problems like them.

CL 4-113. Evan is trying to save \$60 to buy new parts for his bike. He has saved 45% of what he needs so far.

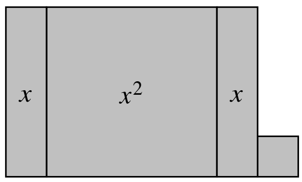
a. Draw a diagram to represent this situation.

b. How much has Evan saved so far?

c. How much does Evan still need to save? Write your answer as a dollar amount and as a percent.

CL 4-114. Mrs. Chen has two brothers. Mark is 7 years older than Mrs. Chen and Eric is 11 years younger than Mrs. Chen. The sum of all three of their ages is 149. Use the 5-D Process to determine the age of Mrs. Chen.

CL 4-115. Sketch the algebra tile shape at right on your paper.

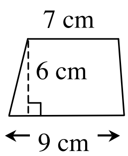


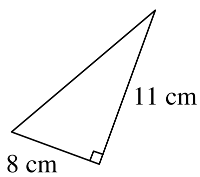
a. Write and simplify an expression for the perimeter of the shape.

b. Evaluate your expression if $x = 5.5$.

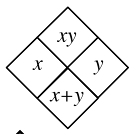
c. What is x if the perimeter is 34?

CL 4-116. Find the area of the trapezoid and the triangle at right. Which figure has the larger area? Explain how you know.

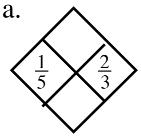




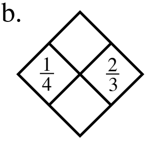
CL 4-117. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



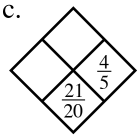
a.



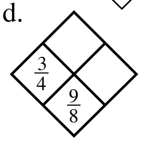
b.



c.



d.



CL 4-118. Copy the following expressions on your paper and simplify them by combining like terms. Using algebra tiles may be helpful.

a. $4x+2+2x+x^2+x$

b. $10x+4-3+8x+2$

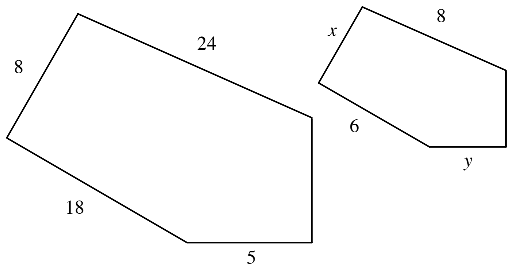
c. $4+x^2+3x+2x^2+4$

d. $x+4+(x-1)+3+2x$

CL 4-119. The shapes at right are similar.

a. What is the scale factor?

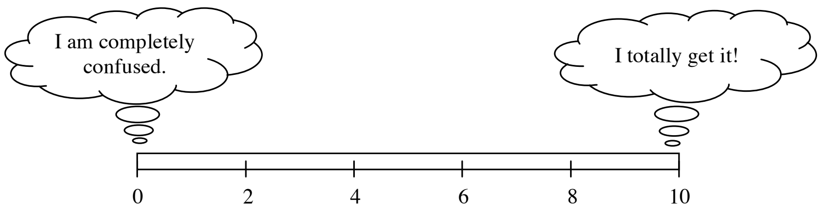
b. Find the sides labeled x and y.



CL 4-120. Over the summer, Gabriel read books that had 192, 202, 175, 219, and 197 pages. What was the average number of pages in the books he read? Show your work.

CL 4-121. For each of the problems above, do the following:

- Draw a bar or number line that represents 0 to 10.



- Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose *one* of those problems and do one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.

③ WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Log, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it a useful reference for you as you complete this chapter and prepare to begin the next one.

Learning Log Entries

- Lesson 4.2.1 – Measures of Central Tendency
- Lesson 4.2.3 – Box-and-Whisker Plot
- Lesson 4.3.2 – Finding Similar Shapes
- Lesson 4.3.4 – Finding Missing Sides of Similar Shapes

Math Notes

- Lesson 4.1.1 – Equivalent Ratios
- Lesson 4.1.2 – Part to Whole Relationships
- Lesson 4.2.2 – Histograms and Bar Graphs
- Lesson 4.2.3 – Quartiles
- Lesson 4.2.4 – Box-and-Whisker Plots
- Lesson 4.3.1 – Corresponding Parts of Similar Shapes
- Lesson 4.3.4 – Scale Factor

Mathematical Vocabulary

The following is a list of vocabulary found in this chapter. Some of the words have been seen in the previous chapter. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also want to add these words to your Toolkit for a way to reference them in the future.

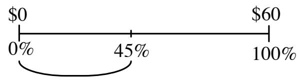
bin	**box-and-whisker plot**	**congruent**
corresponding	histogram	**lower quartile**
median	mode	**partition**
range	**ratio**	**scale factor**
similar figure	**stem-and-leaf plot**	**upper quartile**

Process Words

The list of words below are problem solving strategies and processes that you have been involved in throughout the course of this chapter. Make sure you know what it means to do each of the following. If you are not sure, look through your book for problems when you were asked to think in the following ways.

brainstorm	choose a strategy	describe
explain your reasoning	justify	predict
rearrange	reverse your thinking	test your prediction
visualize		

Answers and Support for Closure Activity #2
What Have I Learned?

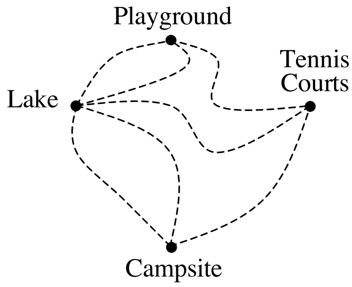
Problem	Solution	Need Help?	More Practice
CL 4-113.	a.  b. $27 c. $33 dollars, or 55% of $60	Lessons 4.1.1 and 4.1.2 Math Notes box in Lesson 4.1.2	Problems 4-2 through 4-10, 4-20, 4-43, 4-67, 4-79, and 4-108
CL 4-114.	Mrs. Chen is 51 years old.	Lessons 3.2.2, through 3.2.5 Math Notes box in Lesson 3.2.3 Learning Log (problem 3-98)	Problems CL 3-113, 4-88, and 4-110
CL 4-115.	a. $P = 4x + 6$ b. 28 units c. $x = 7$	Lessons 3.1.1, 3.1.2, 3.1.3, and 3.1.5 Math Notes boxes in Lessons 3.1.1, 3.1.2, and 3.1.3 Learning Logs (problems 3-28 and 3-37)	Problems CL 3-112, 4-32, 4-60, and 4-102
CL 4-116.	Area of the trapezoid = 48 sq. un. Area of the triangle = 44 sq. un. The trapezoid has the greater area.	Lessons 2.3.4 and 2.3.5 Math Notes box in Lesson 2.3.5 Learning Logs (problems 2-126 and 2-150)	Problems CL 2-163 and CL 3-111
CL 4-117.	a. $\frac{2}{15}$, $\frac{13}{15}$ b. $\frac{2}{12}$ or $\frac{1}{6}$, $\frac{11}{12}$ c. $\frac{4}{20}$ or $\frac{1}{5}$, $\frac{1}{4}$ d. $\frac{9}{32}$, $\frac{3}{8}$	Lessons 1.2.4 and 1.2.5 Math Notes box in Lesson 1.2.4 Learning Log (problem 1-107)	Problems 2-46, 2-57, 2-87, 2-118, 3-63, 4-48, and 4-78

Problem	Solution	Need Help?	More Practice
CL 4-118.	a. x^2+7x+2 b. $18x+3$ c. $3x^2+3x+8$ d. $4x+6$	Lesson 3.1.3 Math Notes box in Lesson 3.1.3 Learning Log (problem 3-28)	Problems 3-32, 3-101, and 4-66
CL 4-119.	a. Divide by 3 or multiply by $\frac{1}{3}$ b. $x=\frac{8}{3}$ or $2\frac{2}{3}$ $y=\frac{5}{3}$ or $1\frac{2}{3}$	Lessons 4.3.1, through 4.3.4 Math Notes boxes in Lessons 4.3.1 and 4.3.4 Learning Logs (problems 4-85 and 4-107)	Problems 4-82, 4-83, 4-84, 4-91 through 4-99, 4-105, and 4-106
CL 4-120.	197 pages	Lessons 1.1.3, 1.1.4, and 4.2.1 Math Notes box in Lesson 1.1.3 Learning Log (problem 4-29)	Problems CL 1-144 and CL 2-157

Puzzle Investigator Problems

PI-7. MAKING TRACKS!

Vu got a new bicycle for her birthday and cannot wait to ride it all around her favorite park. To find out which paths are best, she wants to ride each of them exactly once, without repeating any path and without missing any. When she can do this, it is called an "Euler" (pronounced "oy-ler") path after a mathematician who investigated similar paths.



The map of the park is shown at right. Vu needs to decide where her mother should drop her off to begin her ride.

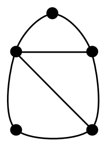
a. Is it possible for Vu to ride each trail exactly once without repeating any path and without missing any?

- If it is possible, show all the possible ways for her to do this.
- If none exist, find a new path you could add to the park to create an Euler path.

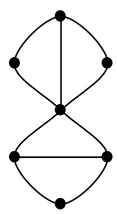
b. The park manager is planning to add a parking lot as a new point on the map. It will need to be connected to at least one of the other locations in the park with a path. Propose a location of a parking lot and at least one other path so that the park will have an Euler path. Remember that to be an Euler path, it must use all the paths exactly once.

c. Vu is thinking about going to one of the parks shown below. Which of them have Euler paths? Which do not? If an Euler path exists, show where Vu could start and stop her ride, and use arrows to show the direction she should travel. Look for reasons why some parks have Euler paths and others do not.

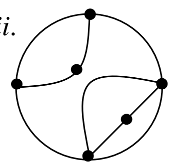
i.



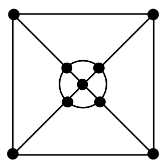
ii.



iii.



iv.

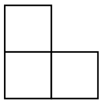


d. Draw two new parks that have Euler paths and two that do not.

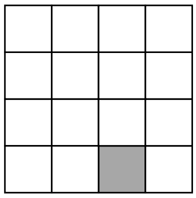
e. Why do some parks have Euler paths and others do not?

PI-8. TILING THE LAUNDRY ROOM

Travis is planning to tile his laundry room with large L-shaped tiles made with 3 squares. (See an example at right.) According to his floor plan, the room is a $4\text{ ft.} \times 4\text{ ft.}$ square. There is a drain in the floor that cannot be covered and is shaded in the diagram.



Tile

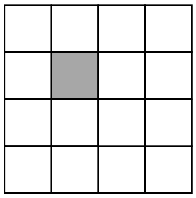


Laundry Room Floor

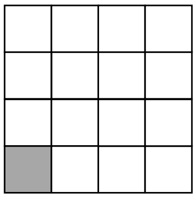
a. Show one way that Travis could tile his floor using his L-shaped tiles without breaking tiles and so that no tiles overlap or cover the drain. Use colors to help distinguish the tiles in your diagram. Is there more than one way to tile his floor?

b. While at the store, Travis suddenly worried that his diagram is wrong and he cannot remember where the drain is located. Does it matter? Can the floor be tiled no matter where the drain is located? Test the different possible locations of the drain (listed below) and write a short note to Travis about what you discovered.

i. Drain is located in the middle



ii. Drain is located in the corner



c. Uh oh! Travis came home with his tiles and found out that his floor is actually a $5\text{ ft.} \times 5\text{ ft.}$ square and the drain is in the corner. (This is why measurements should always be checked twice!) Luckily, he bought extra tiles. However, can this floor be tiled? Using graph paper, draw a diagram of Travis' laundry room floor and find a way he can tile his floor with the same L-shaped tiles.

d. Given that his laundry room is a $5\text{ ft.} \times 5\text{ ft.}$ square, does it matter where the drain is located? Find at least one more location (not in the corner) for the drain that would allow Travis to tile the floor. Also find at least one location for the drain that would not allow the floor to be tiled without breaking a tile.